TU WIEN TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

ATOMINSTITUT

(Im-?)possible prediction of performance change of coated conductors in fusion magnets

Michael Eisterer

Atominstitut, TU Wien, Vienna, Austria

IREF, Gallipoli, June 19th 2025

Outline

Motivation

- Is a prediction of the behavior of a superconducting magnet in a radiation environment possible?
- What is needed?
- Stimulation of the respective discussion at IREF.

How should we aim to predict the changes?

- Experiments? Modelling? A combination of both? Defect structure?
- (Defining benchmarking experiments (validating models), available facilities,…..)

Enhanced pinning versus decreased superfluid density

- Favorable increase in flux pinning („large defects“)
- Harmful suppression of superfluid density („small defects“)

Conclusions

Collaborations and Funding

Raphael Unterrainer, Alexander Bodenseher

David Fischer MIT (proton irradiation)

Daniele Torsello, Francesco Laviano Politecnico di Torino (radiation environment, damage calculations)

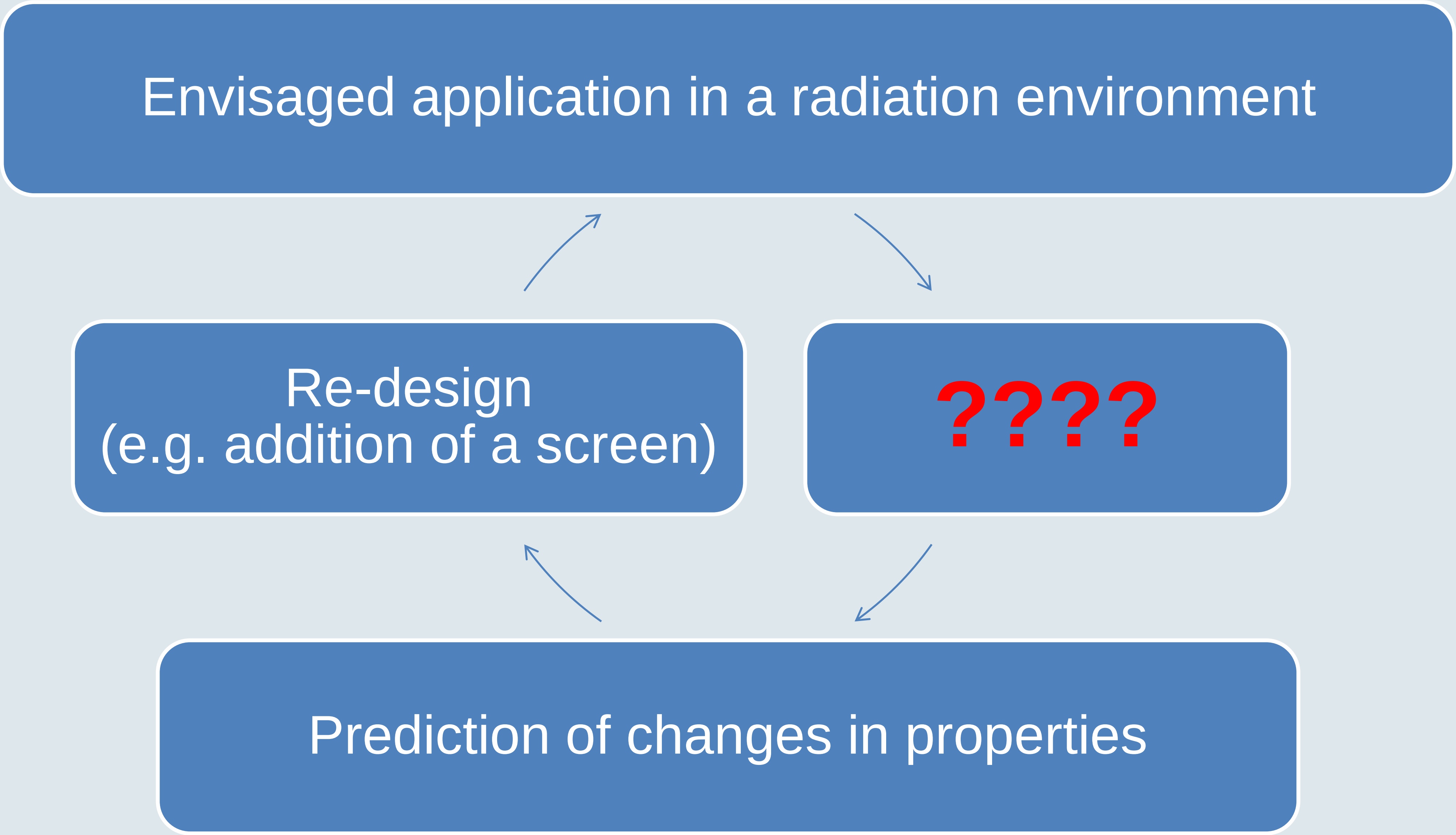
Davide Gambino Linköping University (MD calculations)

Ruben Hühne IFW Dresden (thin film preparation)

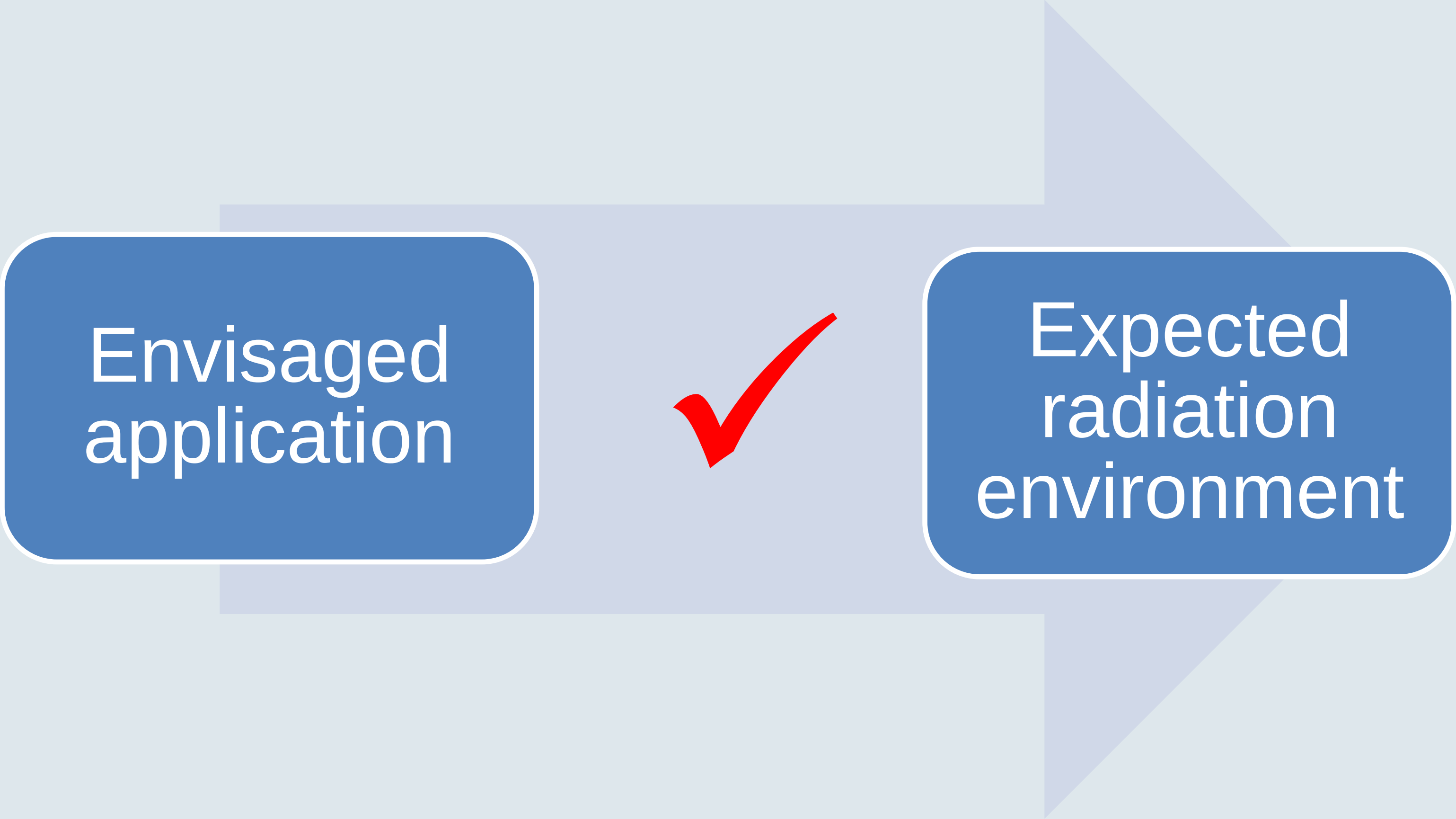
This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

Funded by the European Union

Motivation


Envisaged application in a radiation environment
????
Prediction of changes in properties
Re-design
(e.g. addition of a screen)

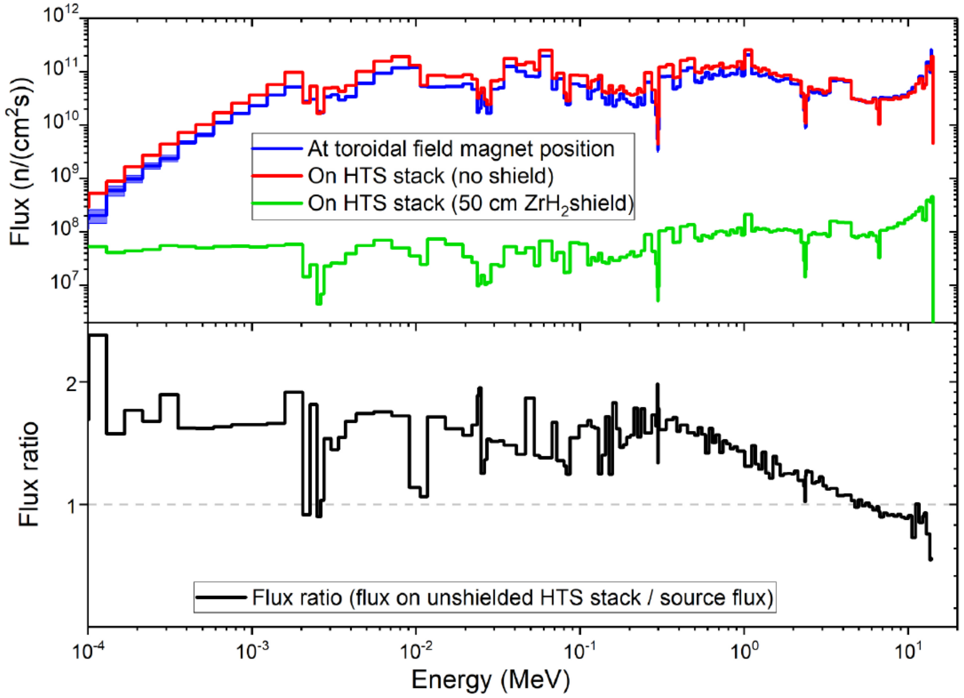
1st step: definition of the radiation field



Sufficient modelling tools available(?)

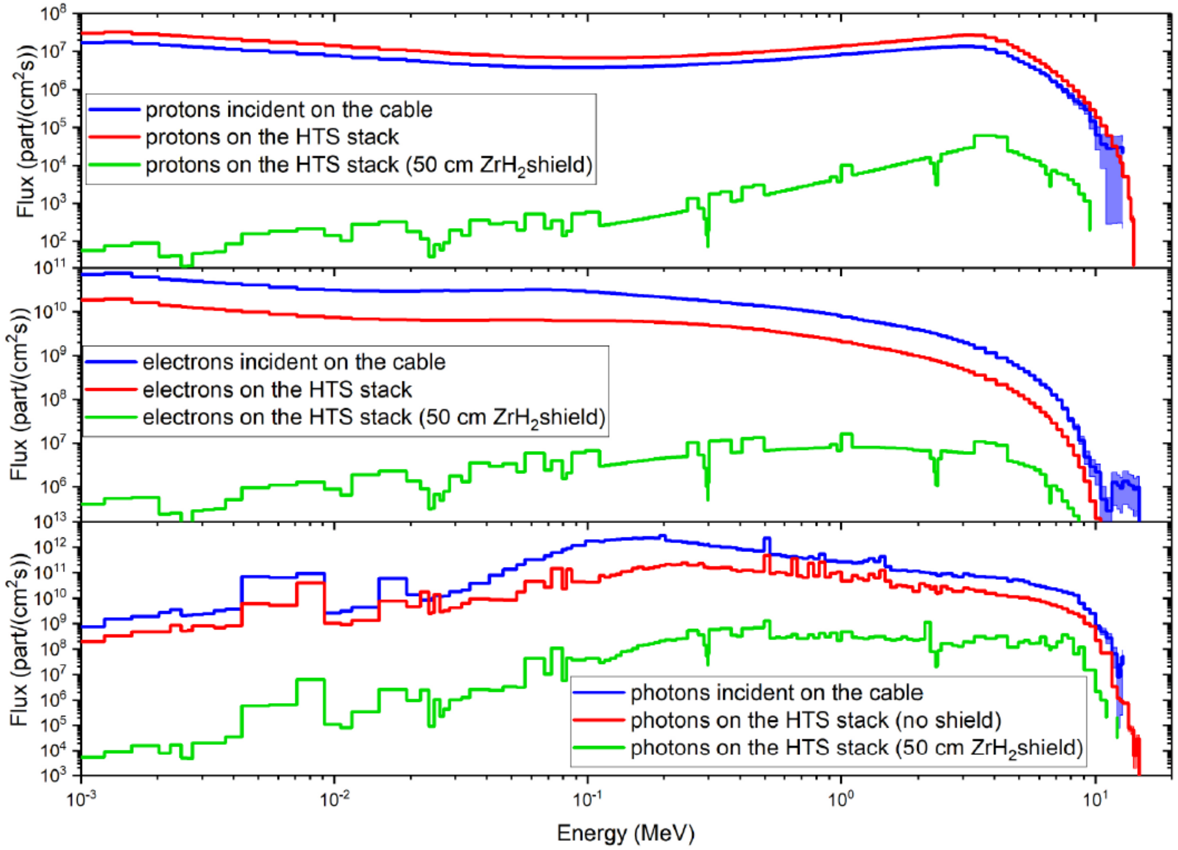
Radiation environment

Neutrons



F. Ledda et al., IEEE TAS **34** (2024) 4206105

Protons, Electrons, Photons



Different particles, energies ranging from ~100 eV to ~15 MeV

Motivation

Predicting the property changes

Expected radiation environment

Experimental

Modelling

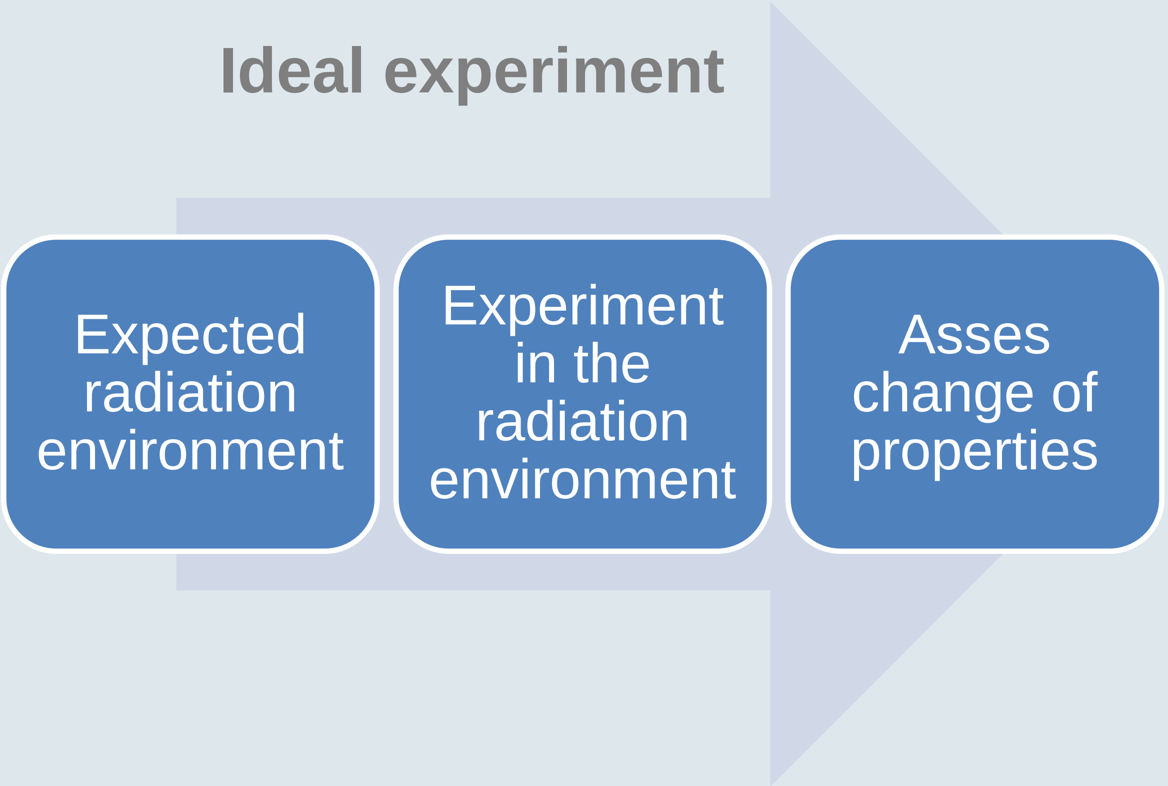
Experimental
+
Modelling

Critical current at operation conditions:

- Field
- Temperature
- Strain
- ….

Motivation

Ideal experiment



Real world: expected radiation environment is not available for experiments.
Much higher flux is required.

→ Finding a proxy is necessary.

→ Similar (?!) defect structure

- All particles (neutrons, protons,…)
- All energies (100 eV to 15 MeV)

Finding a proxy to create a defect structure you don‘t know?

The experimental path

Direct approach

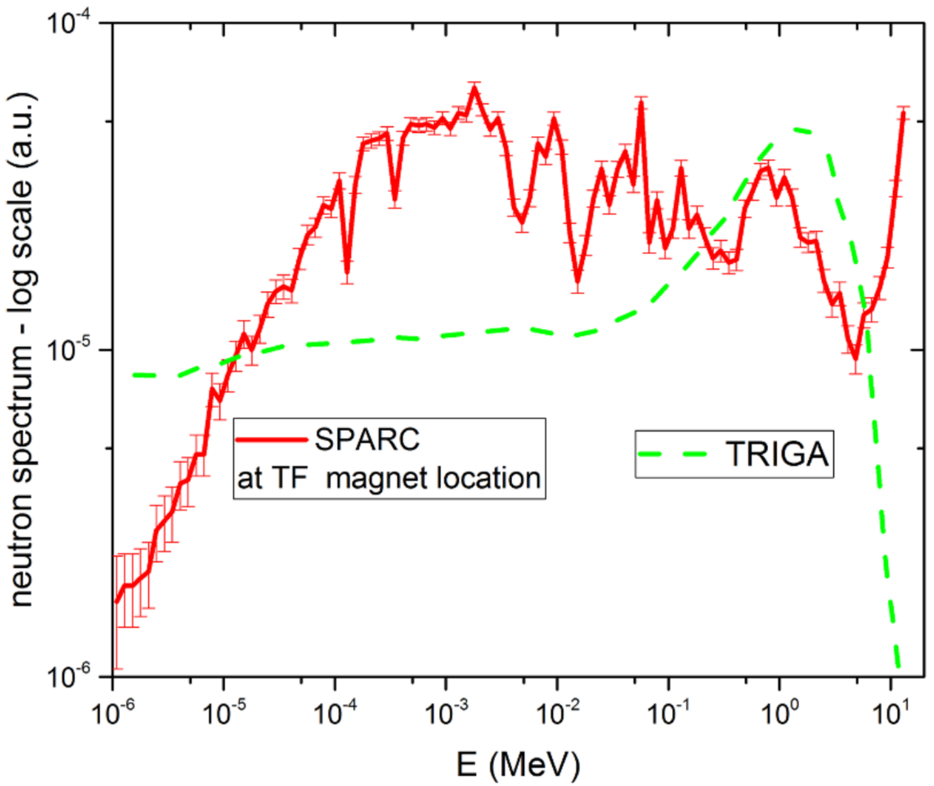
Irradiate the same sample with all particles and at representative energy groups

↓

Measure properties

- Same resulting defect structure as for simultaneous irradiation?
- Keeping the sample cold at all steps?
- Redo for different conductors and particle spectra!

Fast neutrons (likely) don't do the job alone!



D. Torsello et al., SuST **37** (2024) 105008

Finding a proxy

For all relevant particles and representative energies:

Cryogenic irradiation

↓

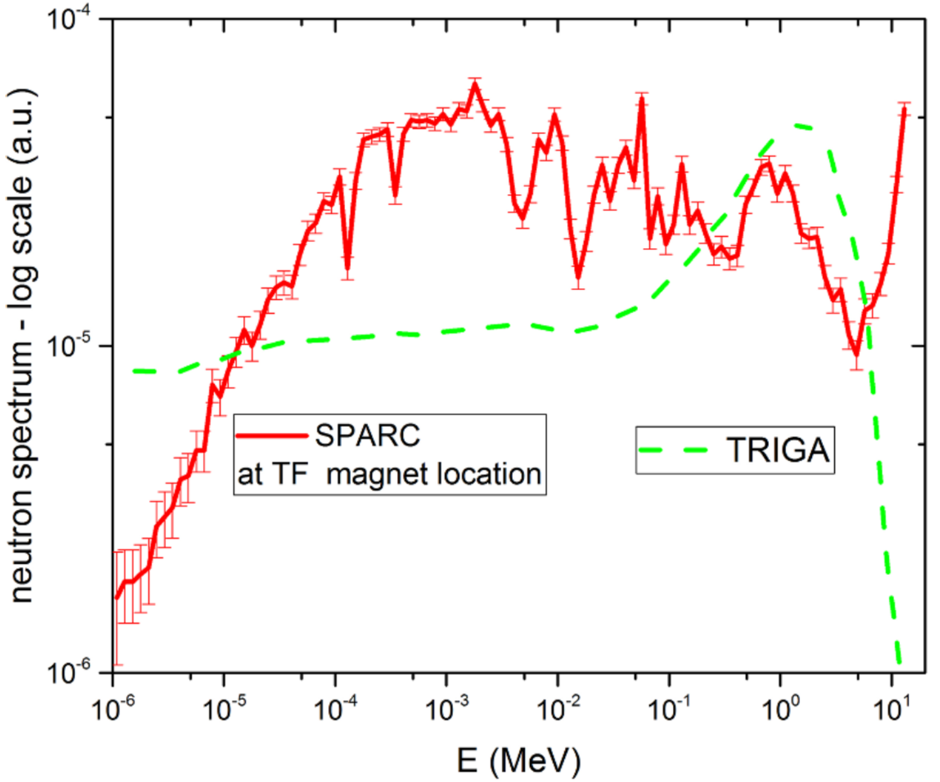
Determination of microstructure
(low temperature, small defects?)

↓

Add the defects according to their contribution

↓

Fast neutrons (likely) don't do the job alone!



D. Torsello et al., SuST **37** (2024) 105008

Chose a combination of particles/energies/fluence as a proxy to mimic this defect structure.

Finding a proxy

For all relevant particles and representative energies:

Cryogenic irradiation

↓

Assess property changes

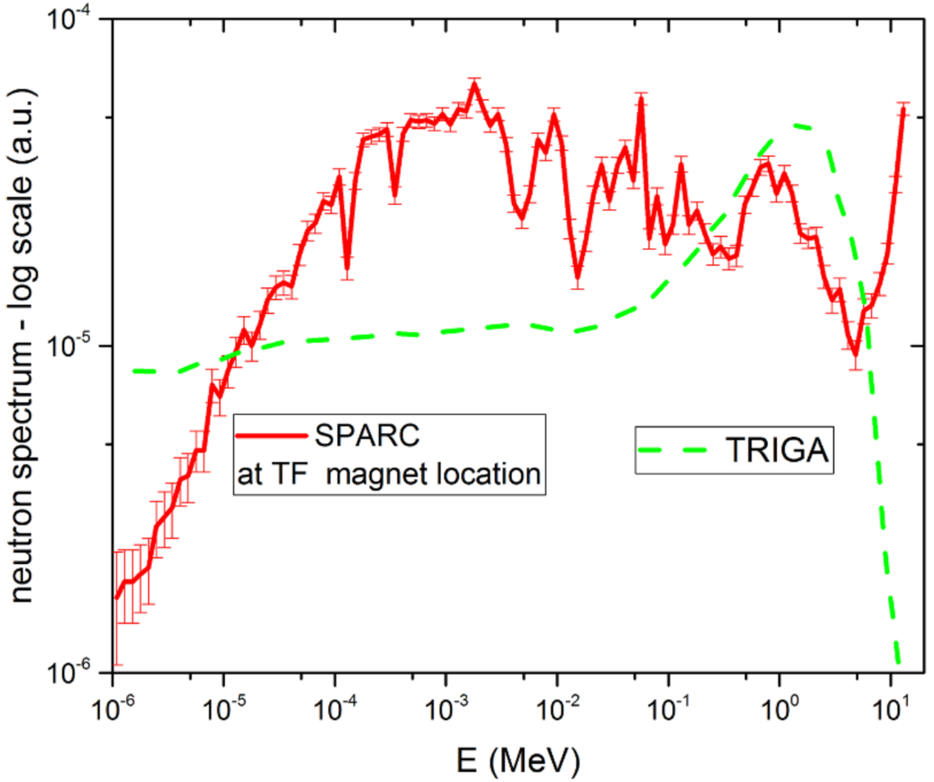
↓

Add the effect of all particles

↓

Chose a combination of particles/energies/fluence as a proxy to mimic these changes.

Fast neutrons (likely) don't do the job alone!



D. Torsello et al., SuST **37** (2024) 105008

Finding a proxy

For all relevant particles and representative energies:

Cryogenic irradiation

↓

Assess property changes

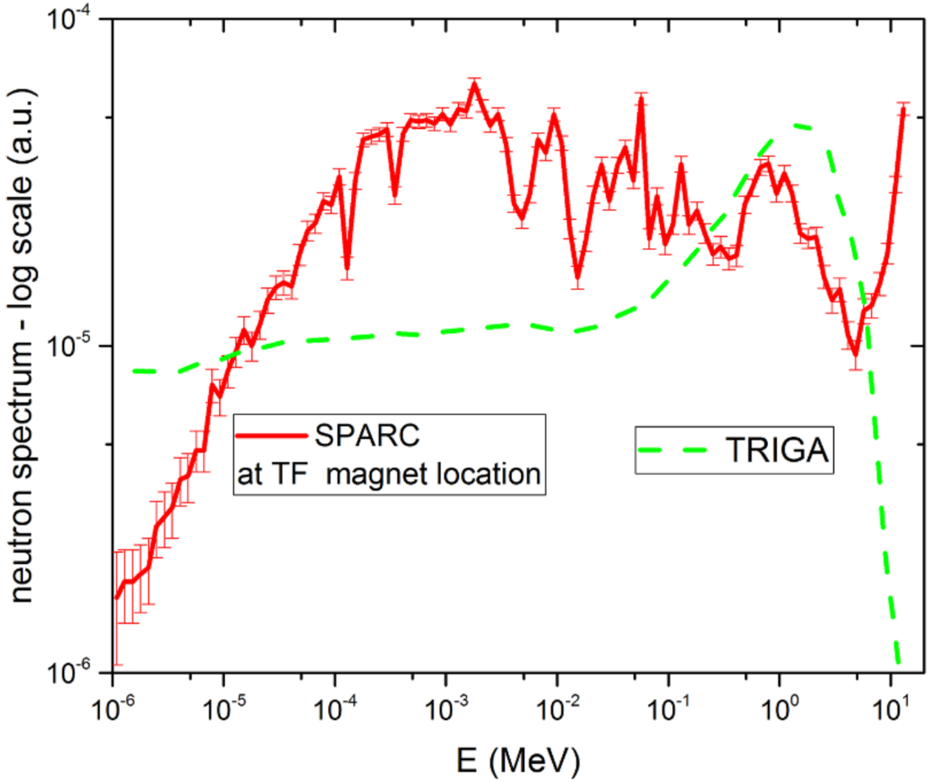
↓

Add the effect of all particles

↓

Chose a combination of particles/energies/fluence as a proxy to mimic these changes.

Fast neutrons (likely) don't do the job alone!



D. Torsello et al., SuST **37** (2024) 105008

The modelling path

Microstructure

Damage (dpa): PHITS, FLUKA ✓?

Defect structure: molecular dynamics ✓?

!!!

Basic superconducting properties

T_c: **???**

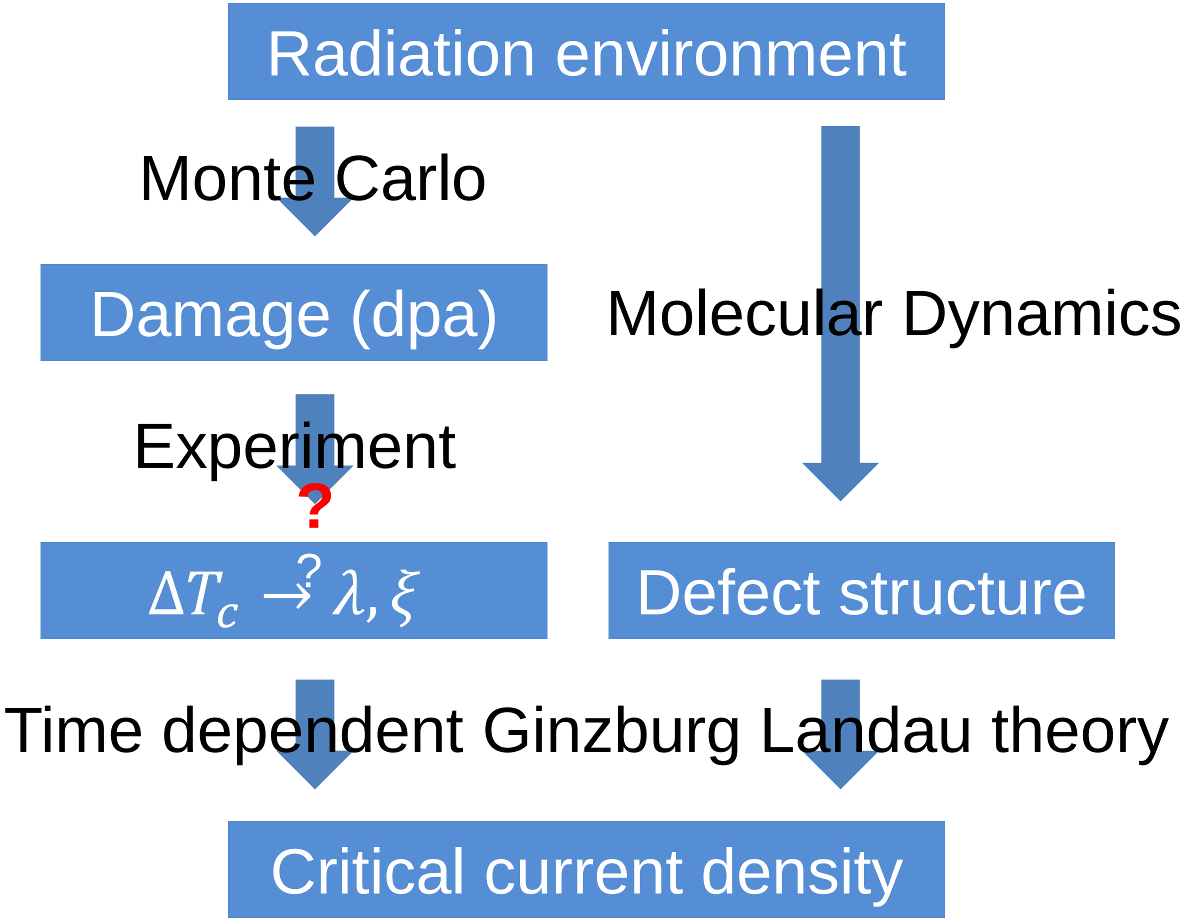
λ, ξ: **???**

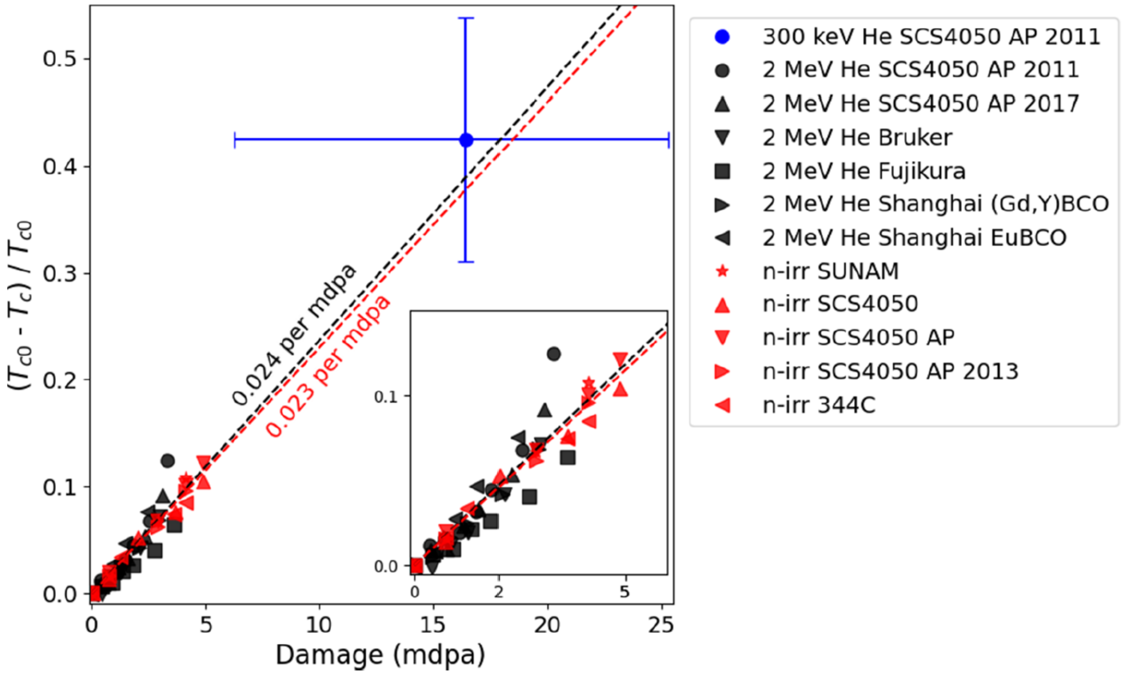
Critical current density

Time dependent Ginzburg Landau theory (TDGL) ✓?

The experimental/modelling path

- Experiments can add missing links e.g. dpa → ΔT_c





K. Adams et al., SuST **36** (2023) 10LT01

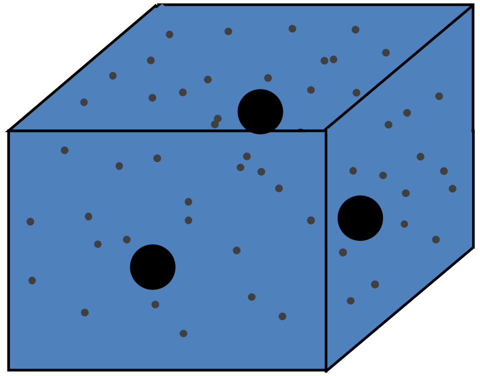
- Fundamental understanding of the underlying physics is required

Disordering matter by radiation

Radiation effects on superconductors

Some definitions

- **Defect**
  - Anything that breaks translation symmetry of the lattice
- **Small defect**
  - Much smaller than the coherence length of the superconductor (~1.5 nm)
  - Voids, interstitials, Frenkel pairs, antisite defects…
  - Do not reduce the order parameter to 0.
- **Large defect**
  - Larger than the coherence length.
  - Non-superconducting volume
- **Disorder**
  - Small defects in the superconducting matrix



Normal state resistivity

Any defect breaks translational symmetry of the crystal lattice
$\rightarrow$ scattering of charge carriers

- Decrease in mean free time τ
- Increase in scattering rate τ^{-1}
- Decrease in mean free path $l = v_{\mathrm{F}}\tau$
- Increase in normal state resistivity $\rho_{\mathrm{n}} = \frac{m_e v_{\mathrm{F}}}{ne^2 l} = \frac{m_e}{ne^2\tau}$

v_{F}… Fermi velocity
m_e… mass of charge carriers
n… density of charge carriers
e… elementary charge

Superconductors

Three important fundamental parameters:

- Transition temperature, T_c
- Magnetic penetration depth, λ:

  Characteristic length for changes of the magnetic field

- Superconducting coherence length, ξ

  Characteristic length for changes of the superconducting order parameter

Other parameters can be calculated from thermodynamics:

Ginzburg-Landau theory:

- Condensation energy density: $E_\mathrm{c} = \frac{\phi_0^2}{16\pi^2\mu_0\lambda^2\xi^2}$
- Upper critical field: $B_{c2} = \frac{\phi_0}{2\pi\xi^2}$

Relation to basic material properties

London theory (basic **electrodynamics** for $\rho = 0$)

Shielding of an applied field, B_a, because of induction

$$B = B_a e^{-\frac{x}{\lambda_L}},\ \lambda_L = \sqrt{\frac{m_e}{\mu_0 n e^2}}$$ (clean superconductors)

General: superfluid density $n_s \propto \frac{m_e}{\lambda^2}$, $\quad \lambda = \sqrt{\frac{m_e}{\mu_0 n_s e^2}}$

Uncertainty principle

Space-Momentum: $\delta x \delta p \geq \frac{\hbar}{2}$, $\delta x \delta k \geq \frac{1}{2}$

$$v_g = \frac{\partial \omega}{\partial k} = \frac{1}{\hbar}\frac{\partial E}{\partial k} \rightarrow \delta k = \frac{\partial k}{\partial E}\delta E = \frac{\delta E}{\hbar v_g} \rightarrow \delta x \geq \frac{\hbar v_\mathrm{F}}{4\Delta}$$

Energy-Time: $\delta E \delta t \geq \frac{\hbar}{2}$: $\tau_{sc} \geq \frac{\hbar}{4\Delta} \rightarrow \delta x \approx v_F \tau_{sc} = \frac{\hbar v_\mathrm{F}}{4\Delta}$

(τ shortend by scattering)

BCS theory

$$\xi_0 = \frac{\hbar v_\mathrm{F}}{\pi \Delta} = 0.18 \frac{\hbar v_\mathrm{F}}{k_B T_\mathrm{c}}$$

$$\Delta = 1.764 T_c$$

Thermodynamics

Energy needed to excite a superconducting electron: $2\Delta \approx k_B T_c \rightarrow \delta x \approx \xi_0 \propto \frac{v_F}{T_c}$

(experimental values available)

Scattering in conventional (s-wave) superconductors

(Non-magnetic) Scattering is not pair breaking in isotropic conventional superconductors.

→ Transition temperature does not change.

→ Condensation energy: $E_\mathrm{c} = \frac{\phi_0^2}{16\pi^2\mu_0\lambda^2\xi^2}$ does not change.

- Superconducting coherence length decreases: $\xi = \frac{\xi_0}{\sqrt{1+\frac{\xi_0}{l}}}$ $\left(\approx \sqrt{\xi_0 l}\right)$
- Magnetic penetration depth increases: $\lambda = \sqrt{\frac{m_e}{\mu_0 n_s e^2}} = \lambda_\mathrm{L}\sqrt{1+\frac{\xi_0}{l}}$

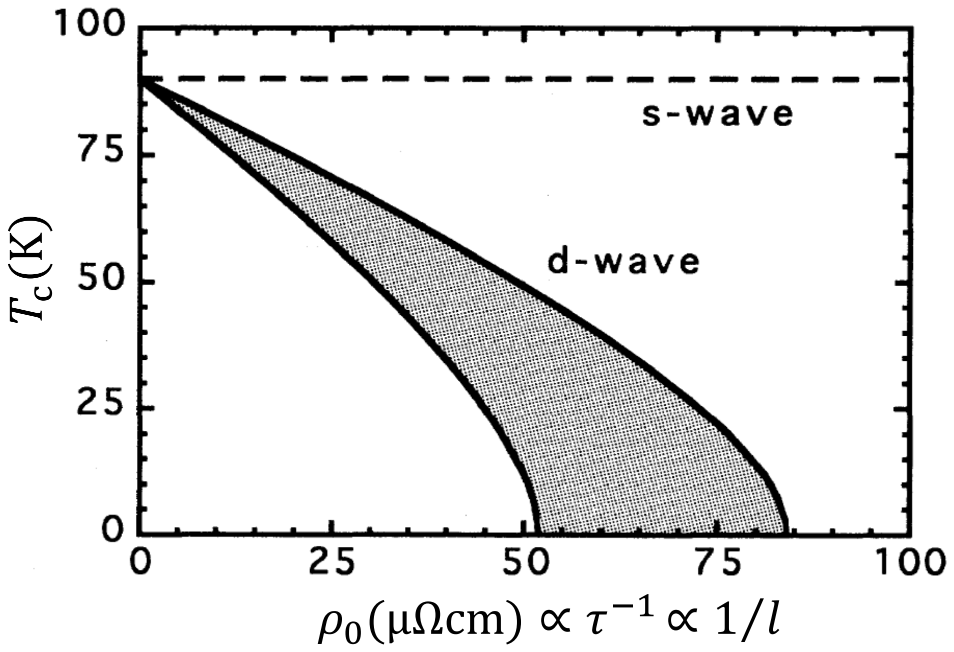
→ Superfluid density $n_\mathrm{s} \propto \frac{1}{\lambda^2}$ is reduced.

Impurity parameter $\alpha := \frac{\xi_0}{l}$, clean limit: $\alpha \ll 1$, dirty limit: $\alpha \gg 1$

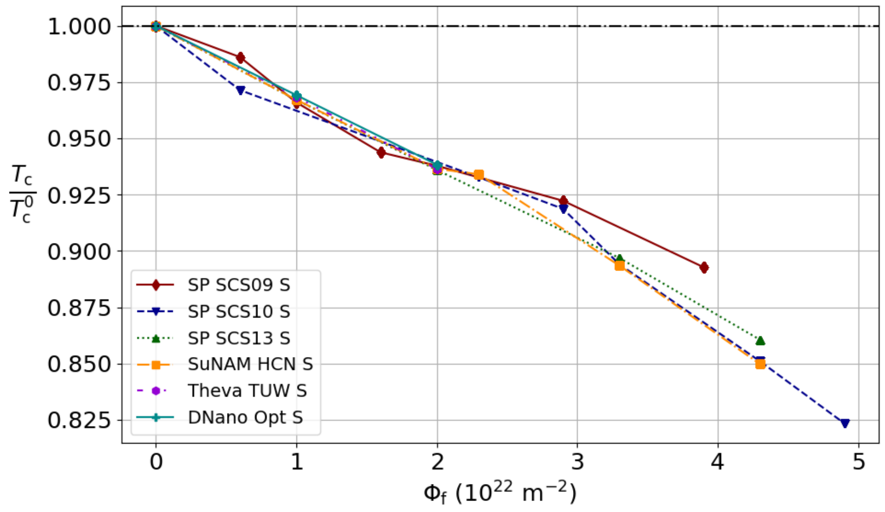
- Depairing current density decreases: $J_\mathrm{d}(D) = \frac{\phi_0}{3\sqrt{3}\mu_0\pi\lambda_\mathrm{L}^2\xi_0\sqrt{1+\alpha}}$

Pair breaking in cuprates

- Scattering is pair breaking in cuprates.
- T_c degrades with increasing resistivity.



R. J. Radtke et al., PRB **48** (1993) 653



R. Unterrainer et al., SuST **37** (2024) 105008

- Resistivity not easily accessible in coated conductors
- Disorder parameter, D: decrease of T_c $(D = T_c^0 - T_c)$
- $D \propto \tau^{-1}$
- $D \propto \phi$ (different slope for different particles)

Scattering in cuprates

- BCS coherence length: $\xi_0 = 0.15 \frac{\hbar v_F}{k_B T_c}$ (d-wave) increases.
- $\alpha = \frac{\xi_0}{l}$ increases stronger.
- Change (increase?) of coherence length: $\xi = \frac{\xi_0}{\sqrt{1+\frac{\xi_0}{l}}}$.
- Magnetic penetration depth increases stronger: $\lambda = \sqrt{\frac{m_e}{\mu_0 n_s e^2}} = \lambda_L \sqrt{1 + \frac{\xi_0}{l}}$.
  - → Superfluid density $n_s \propto \frac{1}{\lambda^2} = \frac{1}{1+\alpha}$ is stronger reduced.
  - → Pair breaking current density, $J_d = \frac{\phi_0}{3\sqrt{3}\mu_0 \pi \lambda^2 \xi}$, decreases stronger.
- Decrease of depairing current density with $D(T_c)$: $J_d(D) = \frac{\phi_0}{3\sqrt{3}\mu_0 \pi \lambda_L^2 \xi_0(D)\sqrt{1+\alpha(D)}}$

Change in superfluid density

$$\frac{n_s}{n_s^p} = \frac{\lambda_p^2}{\lambda^2} = \frac{1+\alpha_p}{1+\alpha} \approx \frac{\alpha_p}{\alpha} = \frac{\xi_0^p l}{l_p \xi_0} = \frac{\rho_n^p T_c}{T_c^p \rho_n}$$

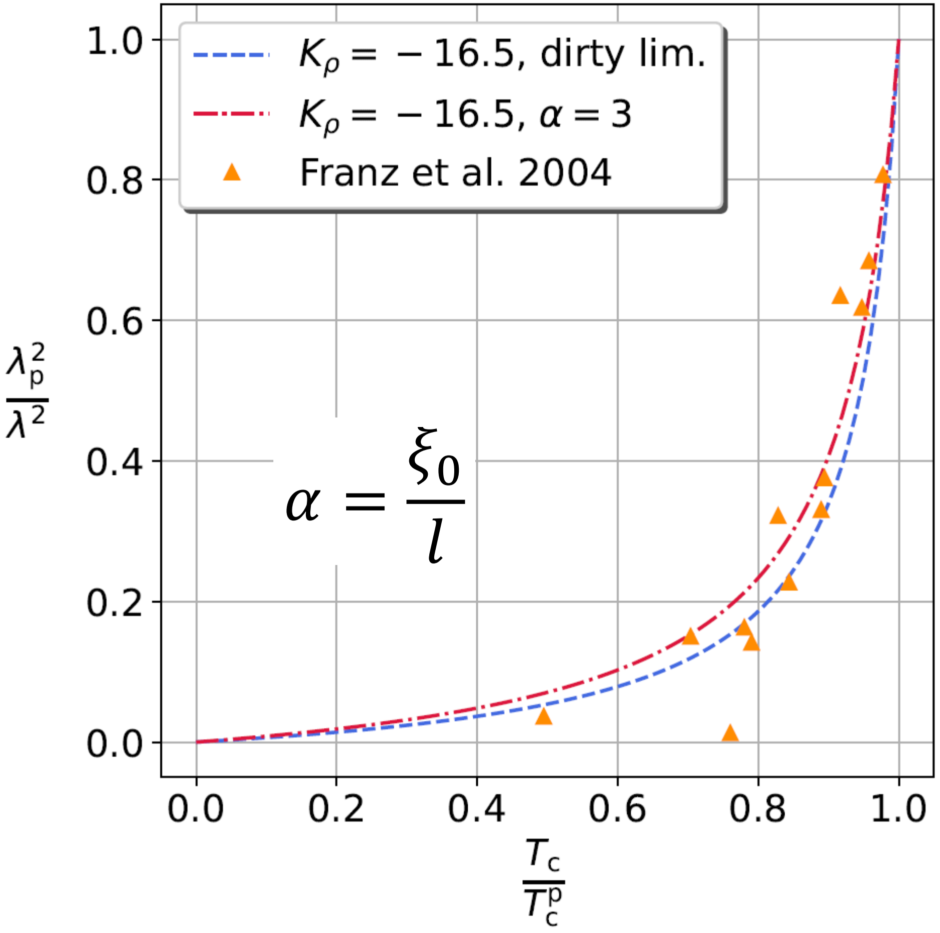
Our experimental data:

$$T_c = \boldsymbol{T_c^p} - \boldsymbol{\beta}\phi,\ \rho_n = \boldsymbol{\rho_n^p} + \boldsymbol{\gamma}\phi$$

$$K_\rho^{-1} = -\frac{\boldsymbol{\beta\rho_n^p}}{\boldsymbol{\gamma T_c^p}} = -\frac{\boldsymbol{\rho_n^p}}{\boldsymbol{T_c^p}} \frac{\partial T_c}{\partial \rho}$$

$\frac{\partial T_c}{\partial \rho}$ universal for optimally doped RE-123?

- Theory: -(0.7-1.2) K/µΩcm
  R. J. Radtke et al., PRB **48** (1993) 653
- **Fast neutrons, Y-123 films: ~ -0.1 K/µΩcm**
- e^- irrad YBCO sc: ~ -0.35 K/µΩcm
  Rullier-Albenque et al., PRL 91 (2003) 047001
- Zn-doped Y-123 sc: ~ -0.28 K/µΩcm
  T.R. Chen et al., PRL 67 (1991) 2088



Experimental data from literature

M. Franz et al. PRB 56 (1997) 7882

Summary scattering

Any defect decreases the mean free path of the charge carrier, l.

- Normal conductors
  - Increase in normal state resistivity $\rho_\mathrm{n} = \frac{m_e v_\mathrm{F}}{ne^2 l}$
- Superconductors
  - Decrease of coherence length in conventional sc: $\xi = \frac{\xi_0}{\sqrt{1+\frac{\xi_0}{l}}} \approx \sqrt{\xi_0 l}$

    Increase of ξ because of increase of ξ_0 (cuprates)
  - Increase of magnetic penetration depth: $\lambda = \lambda_\mathrm{L}\sqrt{1+\frac{\xi_0}{l}}$

  $\rightarrow$ Decrease of superfluid density $\propto \frac{1}{\lambda^2}$

  $\rightarrow$ Decrease of depairing current $\propto \frac{1}{\lambda^2 \xi}$

- Relevant parameter: τ^{-1} ($\rightarrow \rho_\mathrm{n}, T_c$)

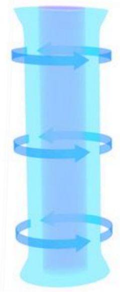
- **Most efficient**: large number of small defects (dpa in superconducting matrix).

Modelling changes in I_c

Flux pinning

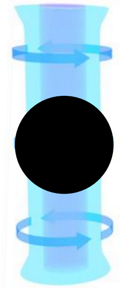
Flux pinning

- Condensation energy density: $E_c = \frac{\phi_0^2}{16\pi^2\mu_0\lambda^2\xi^2}$

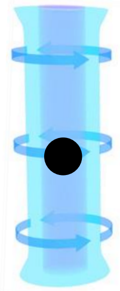


- Energy of vortex core per meter: $E_{core} = E_c\ \pi\xi^2 = \frac{\phi_0^2}{16\pi\mu_0\lambda^2}$

1. Normal conducting/insulating defects (ΔT_c-pinning)



   a. Large defects: $r_D > \xi$: $E_{pin} \cong E_c\ \pi\xi^2 2r_D = \frac{\phi_0^2 r_D}{8\pi\mu_0\lambda^2}$



   b. Small defects: $r_D < \xi$: $E_{pin} = E_c\frac{4\pi r_D^3}{3} = \frac{\phi_0^2 r_D^3}{12\pi\mu_0\lambda^2\xi^2}$

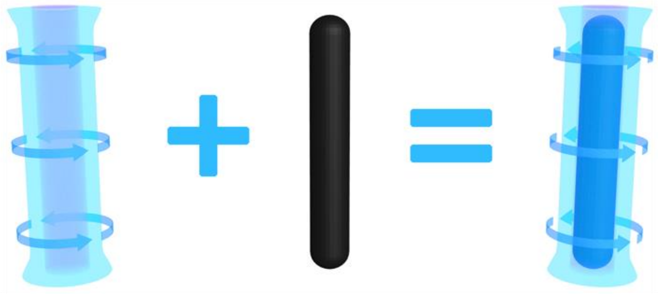
2. Tiny defects, no suppression of E_c (Δl-pinning): vortex core shrinks

- Critical state: $F_p = F_L = |J_c \times B\ |$, force balance, $f_{pin} = \frac{E_{pin}}{\xi}$

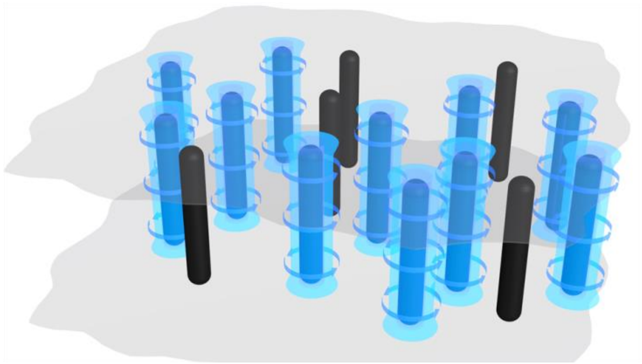
Any defect contributes to pinning!

Pinning efficiency

- Thermodynamic limit: depairing current density
  $$J_\mathrm{d} = \frac{\phi_0}{3\pi\sqrt{3}\mu_0\lambda^2\xi}$$
- Energy of vortex core per meter: $E_\mathrm{core} = \frac{\phi_0^2}{16\pi\mu_0\lambda^2}$
  $$f_p^\mathrm{max} = \frac{E_\mathrm{core}}{\xi} = \frac{\phi_0^2}{16\pi\mu_0\lambda^2\xi}$$



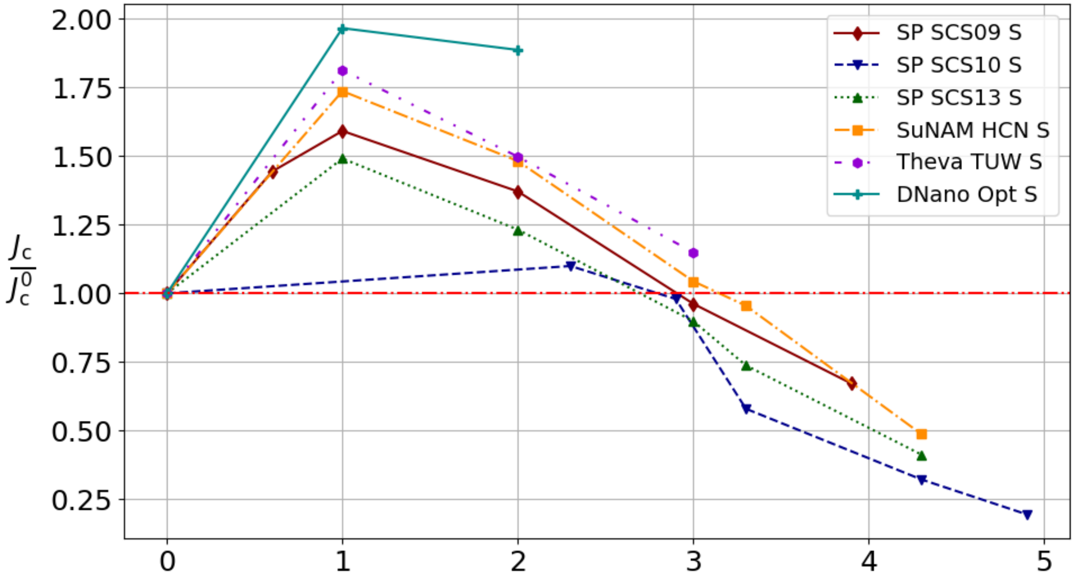
- $J_c^{max} = \frac{f_p^{max}}{\phi_0} = \frac{\phi_0}{16\pi\mu_0\lambda^2\xi} = \frac{3\sqrt{3}}{16}J_d \approx 0.32 J_d$
- $\eta_\mathrm{pin} = \frac{J_c}{J_d}$ ... pinning efficiency
- $\eta_\mathrm{pin,max} \approx 32\%$



- Large defects $r_D \geq \xi$ needed for a large η_pin (although any defect can contribute to pinning)

Change in critical current density

***RE*BCO**
(transport measurements)



Fast neutron fluence (10^{22} m^{-2})

R. Unterrainer et al.,
SuST **37** (2024) 105008

Improved pinning vs. loss in superfluid density

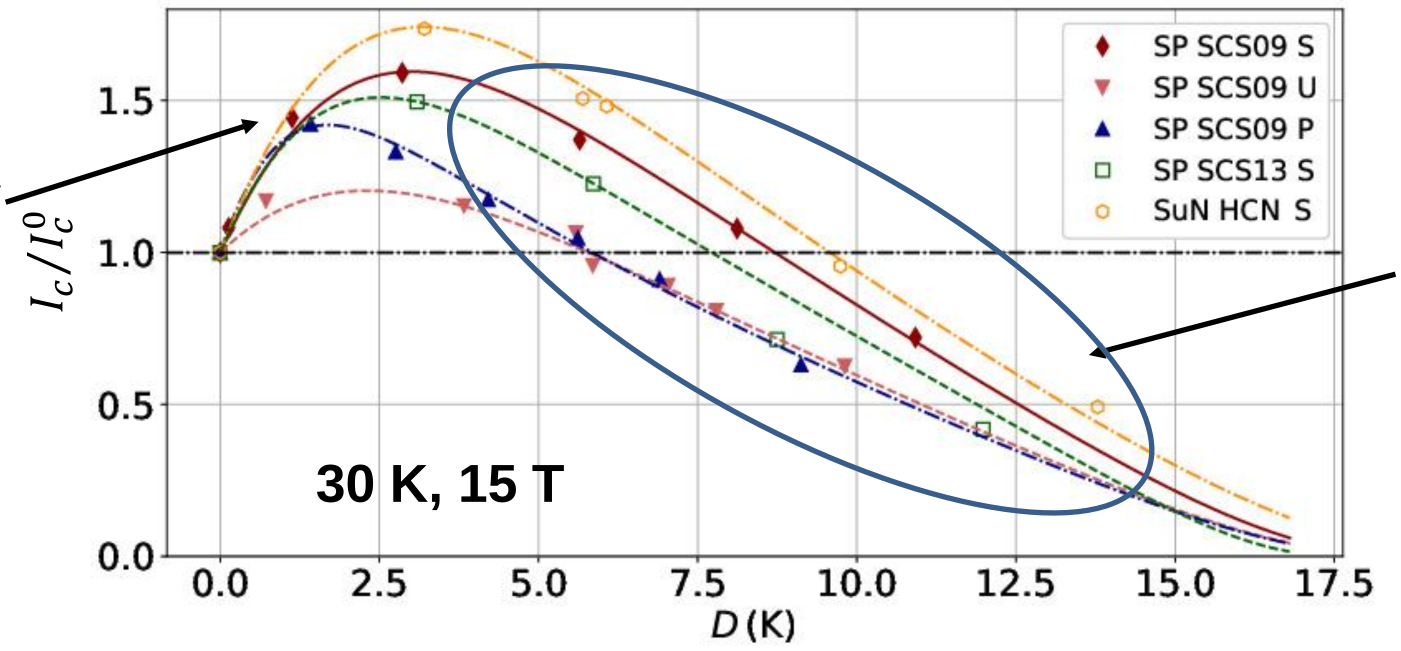
Separation of the two contributions: $J_c = \eta_{pin} J_d(D)$, $\eta_{pin} < 1$……pinning efficiency

Universal degradation

$$J_c = \eta_{pin} J_d(D)$$

Pinning efficiency η_{pin} increases

Large defects?
Good dpa?



J_d decreases

Predictable from changes in T_c and ρ_n.

Small defects?
Bad dpa?

Very similar degradation behavior:

- Same tape (SP SCS09) different irradiation techniques
  - Fast and thermal neutrons (U)
  - Fast neutrons (S)
  - 1.2 MeV protons (P)
- Different tapes (S): SP SCS09, SuN HCN, SP SCS13 (artificial pinning centers)

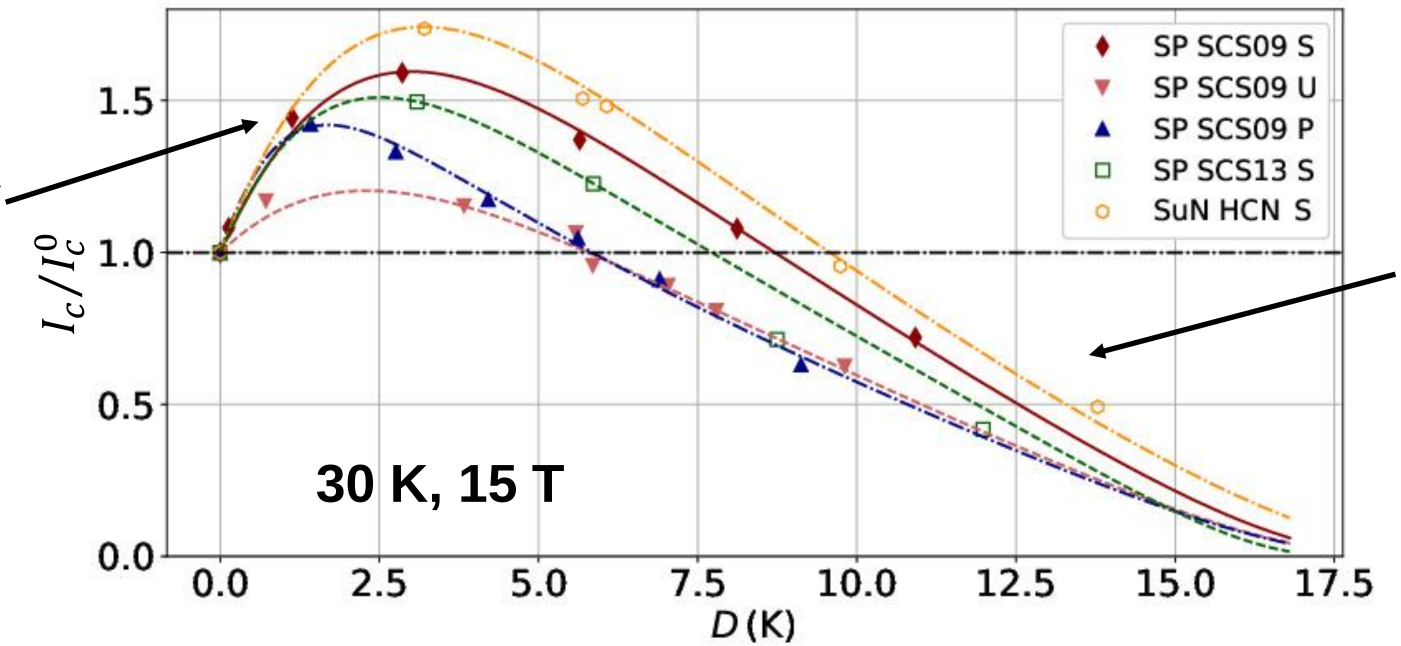
M. Eisterer et al., arXiv:2409.01376v1

Change of critical current

Separation of contributions from enhanced pinning and scattering: $J_\mathrm{c} \propto \eta_\mathrm{pin} J_\mathrm{d}$

Pinning efficiency η_pin increases

Mainly caused by large defects



J_d decreases

Resulting from scattering (small defects)

$$\frac{I_\mathrm{c}}{I_\mathrm{c}^0} = \frac{\eta_\mathrm{pin}}{\eta_\mathrm{pin}^0} \frac{J_\mathrm{d}}{J_\mathrm{d}^0} \frac{A_\mathrm{creep}}{A_\mathrm{creep}^0} =: \frac{\eta_\mathrm{pin}}{\eta_\mathrm{pin}^0} F_\mathrm{D}(D)$$

F_D…degradation function

M. Eisterer et al., arXiv:2409.01376v1

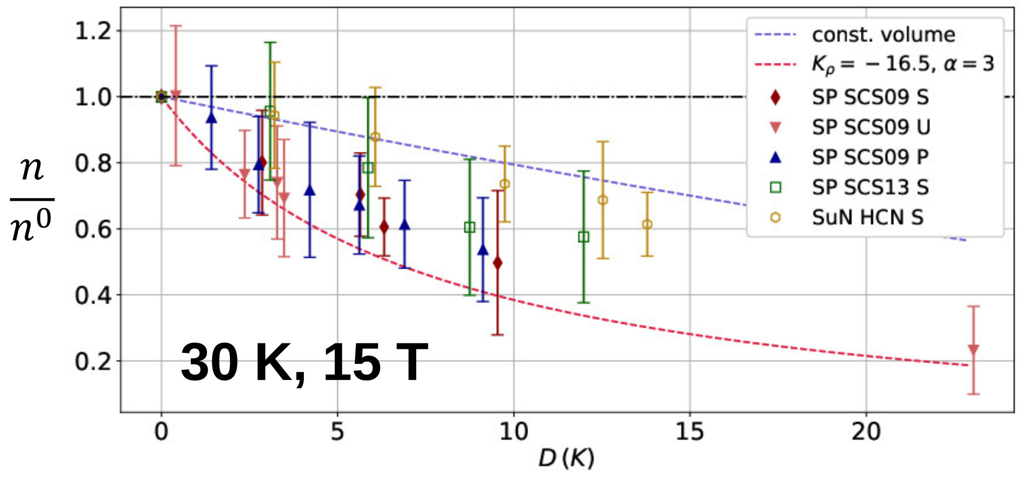
$$F_\mathrm{D} = \sqrt{\frac{(\alpha_\mathrm{p}+1)t_c^3}{\alpha_\mathrm{p}\left(1-K_\rho(1-t_\mathrm{c})\right)+t_\mathrm{c}}} \left(\frac{E_\mathrm{crit}}{\sqrt{\phi_0 B \nu_0}}\right)^{\frac{1}{n}-\frac{1}{n_\mathrm{p}}}$$

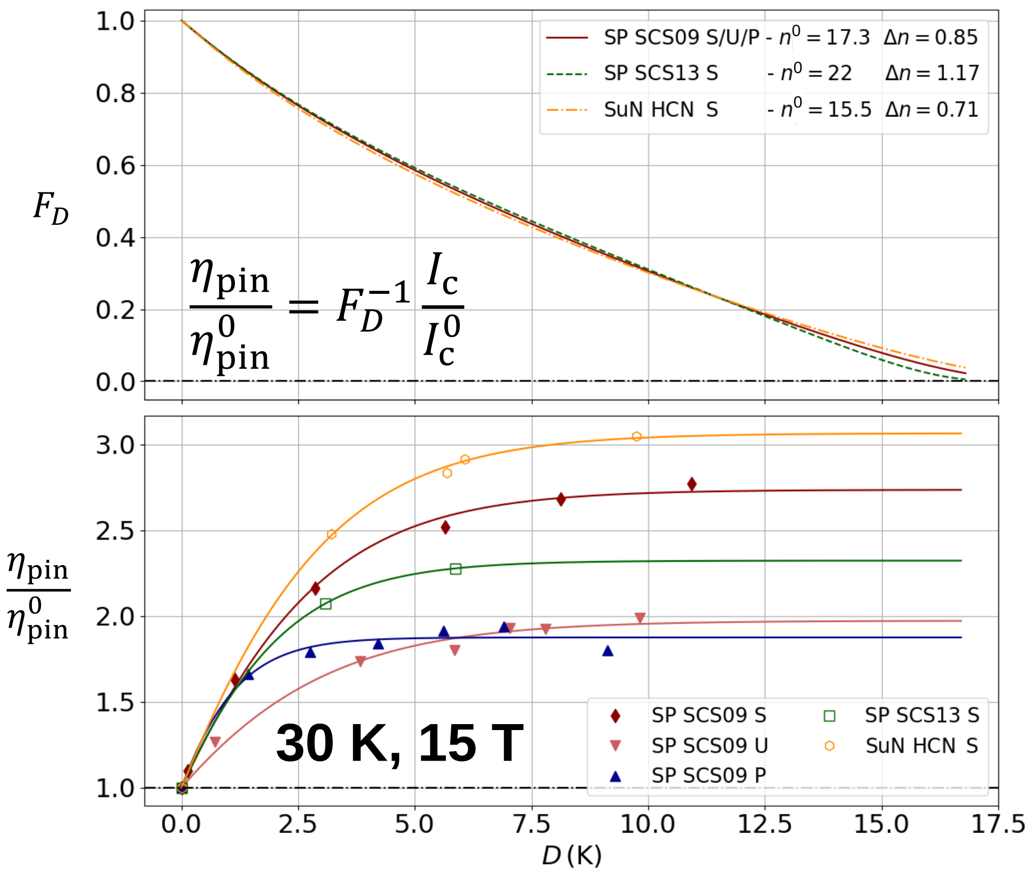
$$t_\mathrm{c} = \frac{T_\mathrm{c}}{T_\mathrm{c,p}} \qquad \alpha_\mathrm{p} = \frac{\xi_\mathrm{0,p}}{l_\mathrm{p}}$$

Degradation function - pinning contribution

Parameters in F_{D}

- $\alpha_{\text{p}} = \frac{\xi_0^0}{l^0}$ (fixed to 3, weak influence)
- $K_\rho = \frac{T_c^0}{\rho_n^0}\frac{\partial \rho_{\text{n}}}{\partial T_{\text{c}}} \approx -16.5$
  (experimental value, thin film)
- n-value, $U \propto I^n$
  linear fit to the experimental values
  (sample dependent)

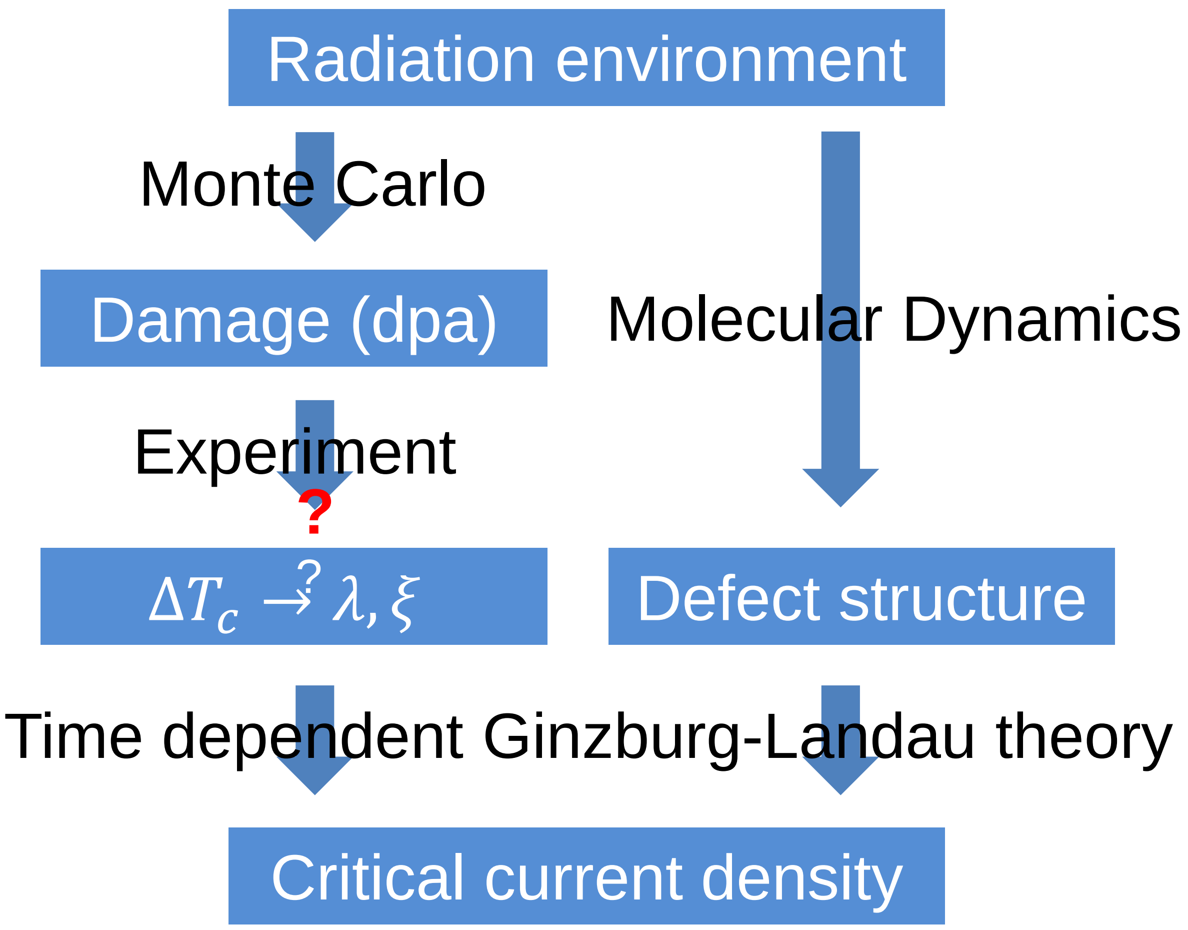




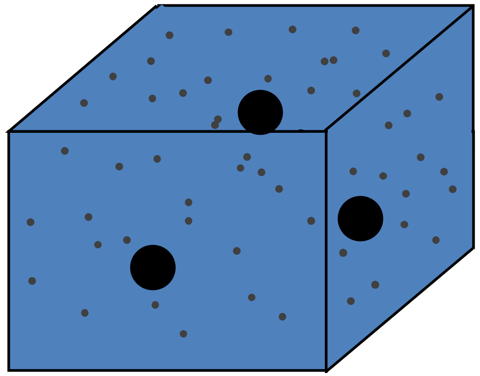
- Strong increase near $D = 0$.
- Saturation at large D.
- $\eta_{\text{pin,max}}$ enough for decreasing branch.

M. Eisterer et al., arXiv:2409.01376v1

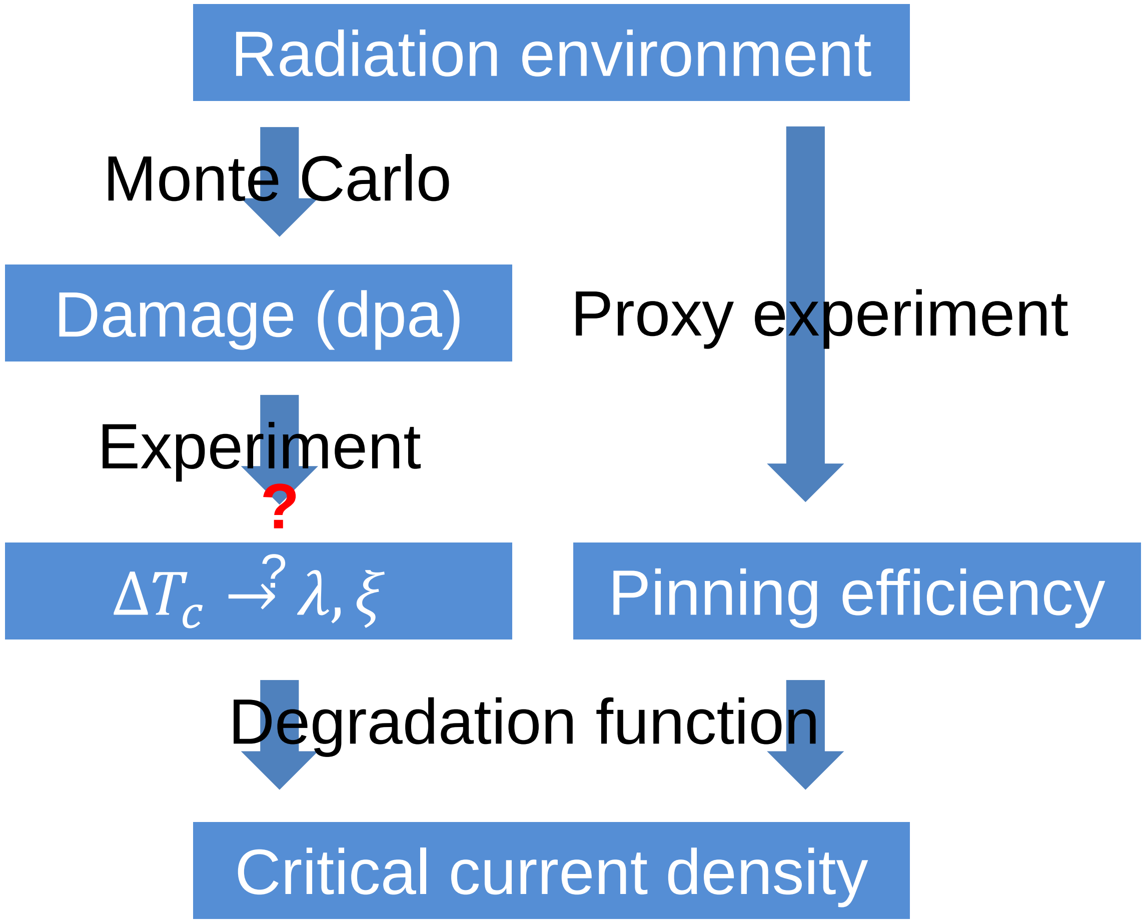
The experimental/modelling path



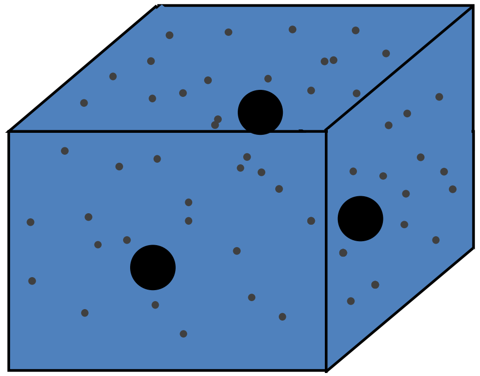
- Scattering rate is the key parameter for the degradation of HTS.
  - Additional parameters?
    - Doping level
    - Strain
- Change in pinning easier to handle?
  - Finding a proxy feasible?
- Damage parameter is needed.
  - Prediction of τ^{-1} or ΔT_c



The experimental/modelling path



- Scattering rate is the key parameter for the degradation of HTS.
  - Additional parameters?
    - Doping level
    - Strain
- Change in pinning easier to handle?
  - Finding a proxy feasible?
- Damage parameter is needed.
  - Prediction of τ^{-1} or ΔT_c



Conclusions

- Predicting the property changes in HTS because of the operation in a specific radiation environment is a difficult task.
  - Competing effects:
    - Degradation due to scattering (small defects)
    - Improvement of pinning (large defects)
- Only a combined approach (experiment+modelling) seems feasible.
- Prediction of scattering rate is key
    - Damage parameter?
- Change of flux creep?
- Change in pinning: Only saturation value of η_{pin} is needed?
    - Benchmarking experiments with proxy particles.

Discussion

I hope you disagree with some points or have some other suggestions so that we have a vivid discussion!

Points to think of

- Prediction of change in flux pinning
- Prediction of change in T_c
- Is it possible to tune the radiation sensitivity?
- Is the change of T_c with resistivity universal? Why not?
- Is $\frac{T_c}{\rho_n}$ a reliable prediction for the (change of) superfluid density?
- What is missing in modelling?
- What is missing from experiments?
- Which benchmarking experiments are missing/needed?

Just in case…..

Scattering in conventional (s-wave) superconductors

(Non-magnetic) Scattering is not pair breaking in isotropic conventional superconductors.

→ Transition temperature does not change.

→ Condensation energy: $E_c = \frac{\phi_0^2}{16\pi^2\mu_0\lambda^2\xi^2}$ does not change.

- Gorkov-Goodman relations: $\kappa = \frac{\lambda}{\xi} = \kappa_0 + 2.37 \cdot 10^6 \sqrt{\gamma_n}\rho_0 = \kappa_0\left(1 + \frac{\xi_0}{l}\right)$

  → Upper critical field increases: $B_{c2} = B_{c2}^{\rho_0 \to 0} + 2.37 \cdot 10^6 \sqrt{\gamma_n}\rho_0 = B_{c2}^{\rho_0 \to 0}\left(1 + \frac{\xi_0}{l}\right)$

  → Superconducting coherence length decreases: $\xi = \frac{\xi_0}{\sqrt{1+\frac{\xi_0}{l}}}$ $\left(\approx \sqrt{\xi_0 l}\right)$

- Isotropic conventional superconductors

  → Magnetic penetration depth increases: $\lambda = \sqrt{\frac{m_e}{\mu_0 n_s e^2}} = \lambda_L\sqrt{1 + \frac{\xi_0}{l}}$

  → Superfluid density $n_s \propto \frac{1}{\lambda^2}$ is reduced.

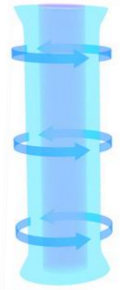
  → Pair breaking current density, $J_d = \frac{\phi_0}{3\sqrt{3}\mu_0\pi\lambda^2\xi}$, decreases.

Modelling changes in I_c

Flux pinning

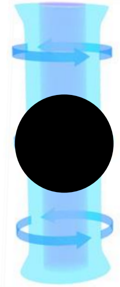
Flux pinning

- Condensation energy density: $E_\mathrm{c} = \frac{\phi_0^2}{16\pi^2\mu_0\lambda^2\xi^2}$

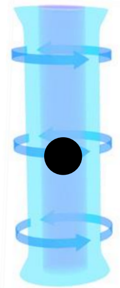


- Energy of vortex core per meter: $E_\mathrm{core} = E_\mathrm{c}\,\pi\xi^2 = \frac{\phi_0^2}{16\pi\mu_0\lambda^2}$

1. Normal conducting/insulating defects (ΔT_c-pinning)



   a. Large defects: $r_D > \xi$: $E_{pin} \cong E_\mathrm{c}\,\pi\xi^2 2r_D = \frac{\phi_0^2 r_D}{8\pi\mu_0\lambda^2}$



   b. Small defects: $r_D < \xi$: $E_{pin} = E_\mathrm{c}\frac{4\pi r_D^3}{3} = \frac{\phi_0^2 r_D^3}{12\pi\mu_0\lambda^2\xi^2}$

2. Tiny defects, no suppression of E_c (Δl-pinning): vortex core shrinks

- Critical state: $F_p = F_L = |J_c \times B\,|$, force balance.
- $f_{pin} = \frac{E_\mathrm{pin}}{\xi}$

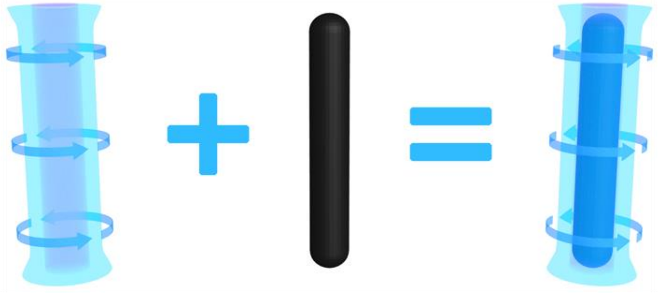
Pinning efficiency

- Thermodynamic limit: depairing current density

$$J_\mathrm{d} = \frac{\phi_0}{3\pi\sqrt{3}\mu_0\lambda^2\xi}$$

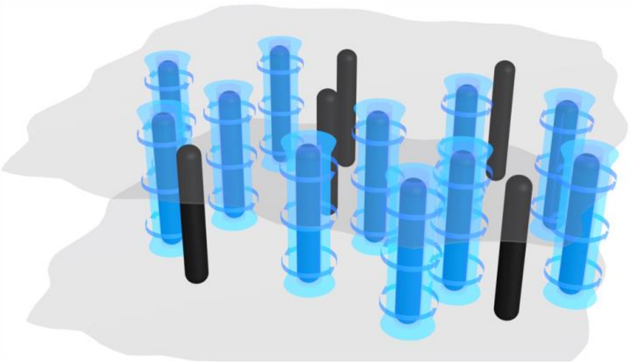
- Energy of vortex core per meter: $E_\mathrm{core} = \frac{\phi_0^2}{16\pi\mu_0\lambda^2}$

$$f_p^\mathrm{max} = \frac{E_\mathrm{core}}{\xi} = \frac{\phi_0^2}{16\pi\mu_0\lambda^2\xi}$$



- Critical state: $F_p = F_L = |J_c \times B\,|$
- Highest possible pinning force per vortex and unit length: cylindrical defect with $r_D \geq \xi$
- Force balance for one vortex ($B \perp J_c$): $f_L = f_p$

$$f_L = \iint F_L \mathrm{d}A = \iint J_c \times B\mathrm{d}A = J_c\phi_0 \leq f_p^\mathrm{max} = \frac{\phi_0^2}{16\pi\mu_0\lambda^2\xi}$$



- $J_c^{max} = \frac{f_p^{max}}{\phi_0} = \frac{\phi_0}{16\pi\mu_0\lambda^2\xi} = \frac{3\sqrt{3}}{16}J_d \approx 0.32 J_d$
- $\eta_\mathrm{pin} = \frac{J_c}{J_d}$ ... pinning efficiency
- $\eta_\mathrm{pin,max} \approx 32\%$
- Large defects $r_D \geq \xi$ needed for a large η_pin (although any defect can contribute to pinning)

Change in superfluid density (Homes's Law)

$\lambda = \lambda_L \sqrt{\frac{\xi_0}{l}}$ with the BCS relation $\xi_0 = 0.18 \frac{\hbar v_f}{k_B T_c}$, $\rho_n = \frac{m_e v_f}{n e^2 l}$ and $\lambda_L = \sqrt{\frac{m_e}{\mu_0 n e^2}}$

$$\frac{1}{\lambda^2 - \lambda_L^2} = \frac{\mu_0 k_B}{0.18\hbar} \frac{T_c}{\rho_n} = \frac{1}{\lambda^2} \quad \frac{\mu_0 k_B}{0.18\hbar} = 9.14 \cdot 10^5\ \Omega\text{m}^{-1}\text{K}^{-1}$$

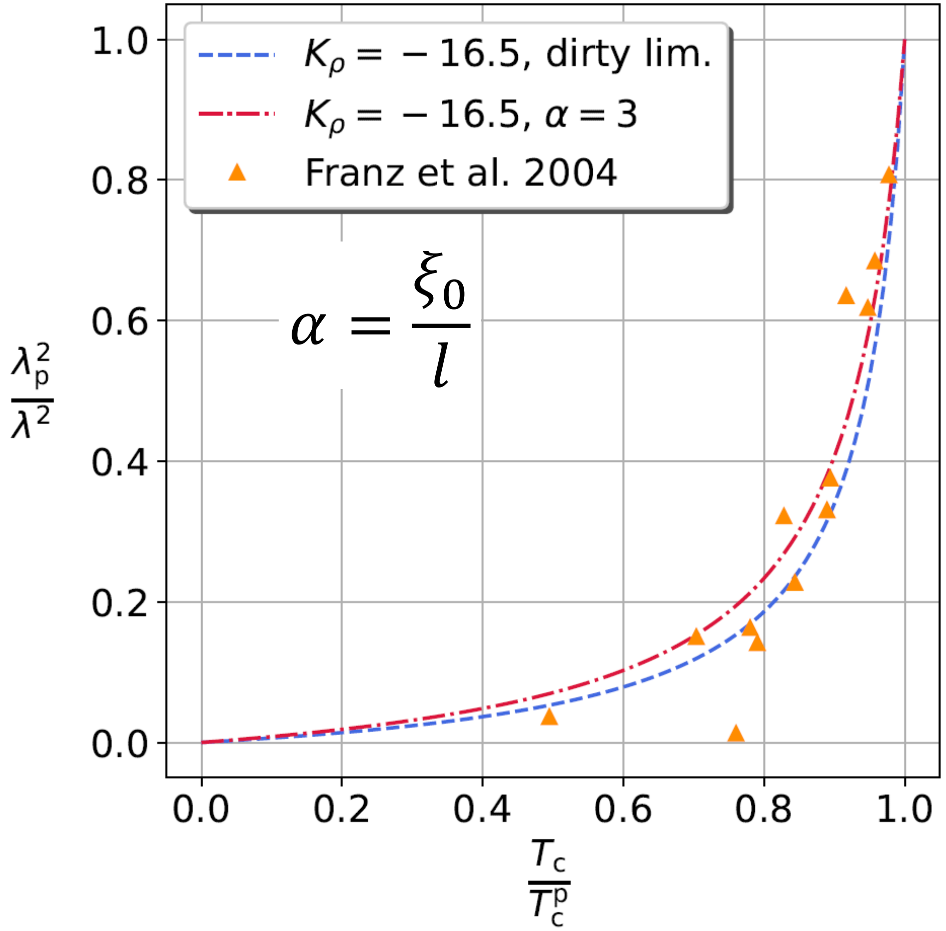
Our experimental data:

$T_c = \boldsymbol{T_c^{unirr}} - \boldsymbol{\beta\phi}$, $\rho = \boldsymbol{\rho_{unirr}} + \boldsymbol{\alpha\phi}$

$$K_\rho = -\frac{\alpha T_c^{unirr}}{\beta \rho_{unirr}} = \frac{T_c^{unirr}}{\rho_{unirr}} \frac{\partial \rho}{\partial T_c}$$

Experimental data from literature:

M. Franz et al. PRB 56 (1997) 7882



Suitable prediction of the change in superfluid density!

Modelling changes in I_c

Resulting changes